Dynamic Tomographic Estimation of Global Exospheric Hydrogen Density and its Response to a Geomagnetic Storm

Gonzalo Cucho-Padin*, Lara Waldrop
ECE Department, Remote Sensing and Space Sciences Group
University of Illinois at Urbana-Champaign.
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dac3@Illinois.edu

Everyone is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid.

- Albert Einstein
Knowledge of exospheric H density is important but conventional estimation techniques are limited.

What is the topic of study?
- Atomic hydrogen (H) located at the outermost layer of the Earth’s atmosphere, resonantly scatters solar Lyman-alpha (121.6nm) radiation

Why do we need to study this topic?
- To understand various solar-terrestrial interactions such as ring current decaying rate, plasmaspheric refilling as well as evaluate the permanent H escape.

How can we measure the H density?
- Direct (in situ) sensing vs. remote sensing.

Main Goal: Generate a remote sensing technique to estimate the Time-dependent, 3-D Hydrogen density distribution in the exosphere.
Hydrogen density estimation leverages the linearity of the optically thin emission model (>3R_E)

\[ I(r, \hat{n}, t) = \frac{g^*(t)}{10^6} \int_{0}^{L_{max}} n_H(l) \Psi(\beta) dl + I_{IP}(\hat{n}, t) \]

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Example of technique feasibility using the NASA’s TWINS mission data (static reconstruction)

- NASA’s Two Wide-angle Imaging Neutral-atom Spectrometers (TWINS) mission provides the capability for **stereoscopically imaging the magnetosphere**.

- Each TWINS1/2 has two **Lyman-alpha detectors (LAD)**, optical sensors.

- The selected data in this study is from **11 June 2008**, in order to compare results with those reported by Bailey et al., [2011]

- Since it is quiet-time we assume a **temporally-static H exosphere**.
Discretization of the exospheric volume of interest yields an algebraic linear system.

\[
I(r_i, \hat{n}_i) = \frac{g^*(r_i)}{10^6} \int_0^{L_{max}} n_H(l) \Psi(\hat{n}_i) dl + I_{IP}(\hat{n}_i)
\]

- **Step 1**: Discretize region into \(J\) spherical voxels.

- **Step 2**: Project unknown density function onto \(J\) orthonormal basis functions.

\[
n_{H}(r') = \sum_{j=1}^{J} x_j \delta_{H_j}(r'),
\]

\[
\delta_{H_j}(r') = \begin{cases} 
1 & \text{if } r' \in V_j \\
0 & \text{else}
\end{cases}
\]

- **Step 3**: Rewrite \(i^{th}\) measurement of intensity as a linear equation.

\[
y(r_i, \hat{n}_i) = \sum_{j=1}^{J} \left[ \frac{g^*(r_i)}{10^6} \Psi(\hat{n}_i) \int_0^{L_{max}} \delta_{H_j}(l) dl \right] x_j
\]

\[
y = Lx
\]

\[
y \in \mathbb{R}^M, \quad x \in \mathbb{R}^J, \quad L \in \mathbb{R}^{M \times J}
\]
Solving the estimation problem requires the use of more complex techniques such as regularization.

- Observation matrix \( L \in \mathbb{R}^{M \times J} \), \( M \gg J \) and is not full column rank (Voxels without LOS through them).

- Regularization techniques are necessary to obtain a solution.

- The selected regularization method is Regularized Robust Positive Estimation.

- Includes prior knowledge of physical structure of the Hydrogen density distributions for each dimension.

\[ \hat{x} = \arg\min_{x \geq 0} \Phi(x) \]

\[ \Phi(x) = ||Lx - y||_2^2 + \lambda RRPE(x) \]

<table>
<thead>
<tr>
<th>Cost Func.</th>
<th>Data misfit term</th>
<th>Regularization term</th>
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\[ \lambda RRPE(x) = \lambda_r ||x||_{D_r} + \lambda_\phi ||x||_{D_\phi} + \lambda_\theta ||x||_{D_\theta} \]

|-------------|----------------|------------|

\[ ||x||_{D_r} = x^T D_r^T D_r x \]

- Discrete matrix form of 1st and 2nd derivatives

\[ D_r \approx \frac{\partial^2}{\partial r^2} \]
\[ D_\phi \approx \frac{\partial}{\partial \phi} \]
\[ D_\theta \approx \frac{\partial}{\partial \theta} \]
Radial Shell  
\( r = 4.125 \text{ Re} \)  

Radial Shell  
\( r = 6.375 \text{ Re} \)

[Cucho-Padin & Waldrop, JGR, 2018]
Space-state framework approach for “dynamic tomography” and Kalman Filter as a solver

As exospheric H densities are prone to be dynamic during storm-time, we use the state-space model as a means for time-varying estimation:

**Measurement equation:**
\[ y_i = H_i x_i + v_i \]

**Model evolution equation:**
\[ x_{i+1} = F_i x_i + u_i \]

**Inclusion of regularization terms**

\[
\begin{bmatrix}
  y_i \\
  0
\end{bmatrix} =
\begin{bmatrix}
  H_i \\
  D_i
\end{bmatrix}
\begin{bmatrix}
  x_i \\
  v_i
\end{bmatrix}
+ \begin{bmatrix}
  v_i \\
  w_i
\end{bmatrix}
\]

\[
y_i' = H_i' x_i + v_i'
\]

\[
R_i' \triangleq \mathbb{E}[v_i'(v_i')^T] =
\begin{bmatrix}
  R_i & 0 \\
  0 & \lambda_i^{-1} I
\end{bmatrix}
\]

**Dynamic tomographic estimation connected to the LMMSE estimation**

\[
\hat{x}_i^d |_{i|i} = \arg\min_{x_i} ||y_i' - H_i' x_i||^2_{R_i'^{-1}} + ||x_i - \hat{x}_{i|i-1}||^2_{P_{i-1}'^{-1}} + \lambda_\phi ||D_\phi x_i||^2_2 + \lambda_\theta ||D_\theta x_i||^2_2 + \lambda_r ||D_r x_i||^2_2
\]
Using KF we have performed 160 dynamic reconstructions during the storm occurred in 15, June, 2008.

Hydrogen density enhancements during the storm development. Such increments are then translate to higher altitudes with certain delay which suggest a vertical transport or upwelling.
Hydrogen density enhancement at 3.2 Re is equal to ~15%.

In the subsolar point, calculations between 3.2Re and 3.9 Re profiles result in a exospheric wind of ~60m/s.

[Cucho-Padin & Waldrop, GRL, 2019]
Dynamic tomography based on TWINS observations shows that H density increases abruptly in response to the geomagnetic storm on 15 June, 2008. The increment rate and its magnitude varying with distance from Earth.

Density increases begin soonest in the innermost exospheric region in the reconstruction (3.2 RE) and reach a peak density fastest there. Overall density enhancements of ~15% are observed at 3.2 RE. Recovery to pre-storm values is very slow.

Also, analysis of the radial structure for the subsolar point yielded a ~60 m/s wind in vertical direction.

Further work:
1. Conduct similar experiments during a strong geomagnetic storm.
2. Use of tomographically-reconstructed H densities in ring current and plasmasphere analysis during storm-time.
Cucho-Padin & Waldrop, *Tomographic Estimation of Exospheric Hydrogen Density Distributions*, Journal or Geophysical Research, 123, 5119–5139, 2018

