Automatic discovery of the equations governing radiation belt dynamics

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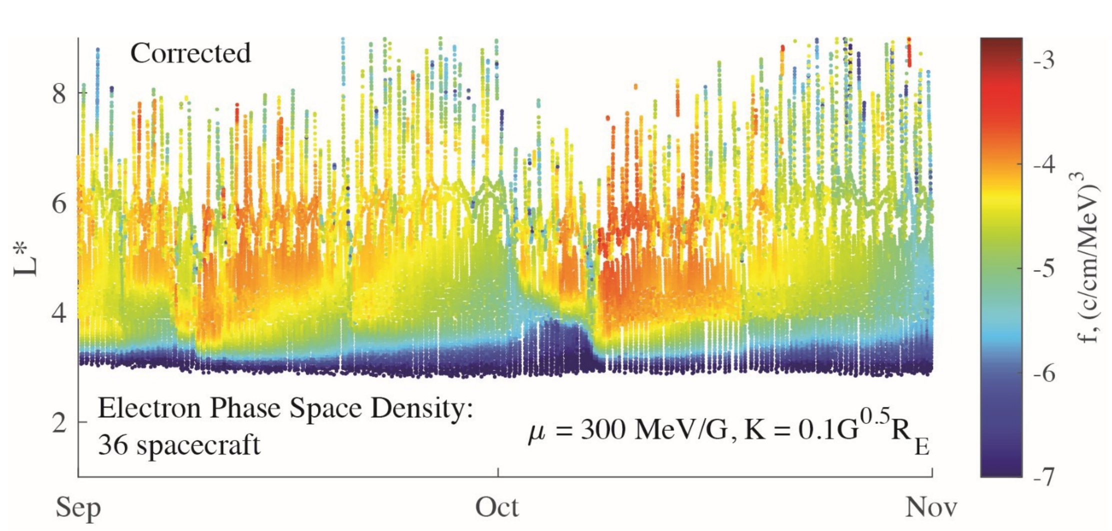
Motivation

- Radiation belt dynamics are typically modeled **by numerically solving the Fokker-Planck diffusion equation.** However, this traditional mode of analysis is heuristic and requires a priory physical understanding and a number of assumptions regarding the form and variation of the diffusion coefficients that determine the rate of transport.
- With the rapid development of satellites, sensors, computational power, and data storage in the past decade, vast quantities of data accumulated at the explosive growth rate now offer new opportunities for **the data-driven discovery of the underlying physics, directly from the data itself.**

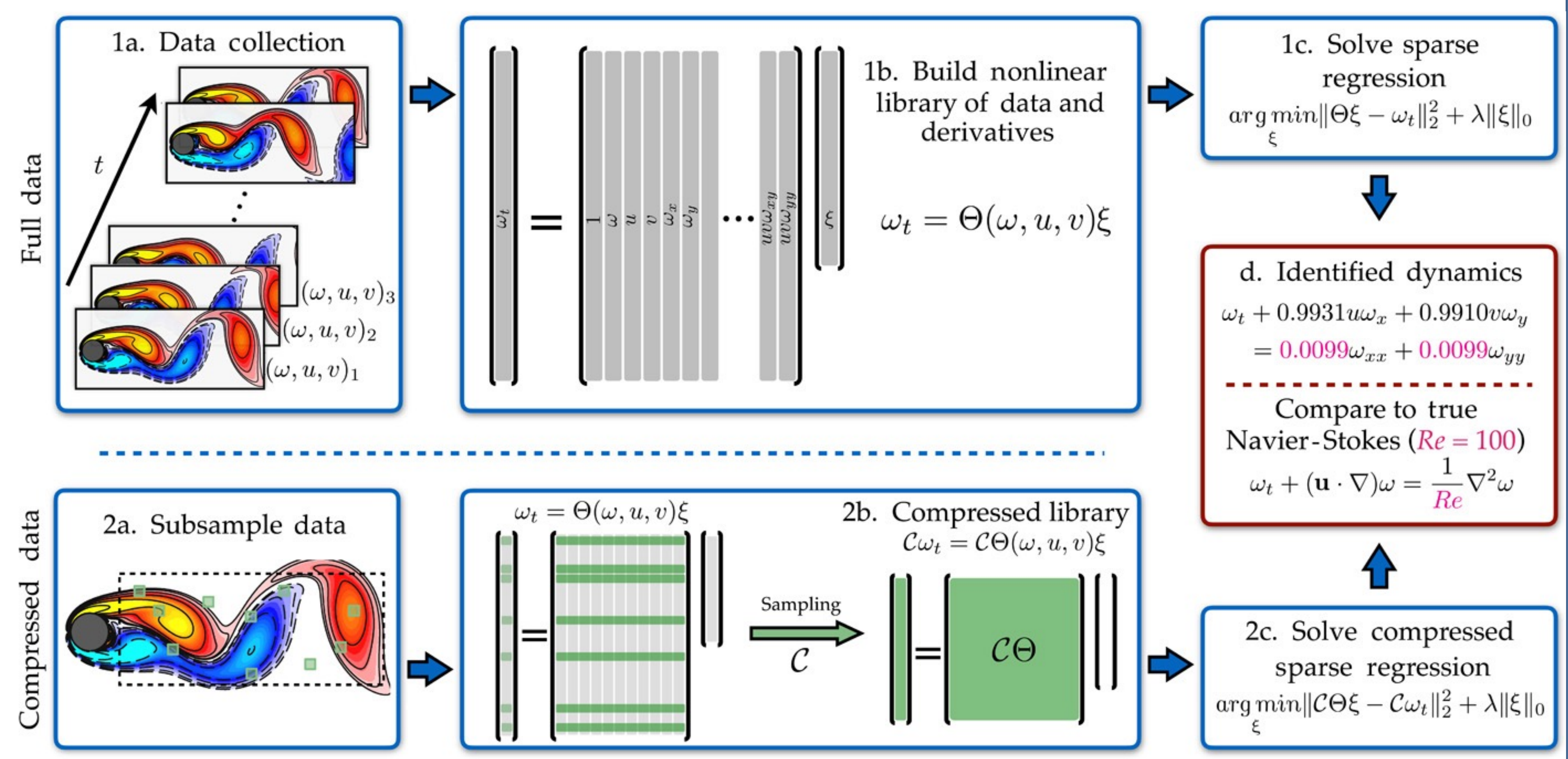
Data

- The PSD data used in this study are intercalibrated by using pairs of spacecrafts, where one spacecraft and instrument are chosen as a gold standard, and the correction is performed for each fixed energy channel on the other spacecraft.
- In this case, Van Allen Probe B and bias-corrected GOES 15 data are used as gold standards to calibrate all the other data.
- We will use machine learning method to reconstruct the PSD model with better resolution



Method

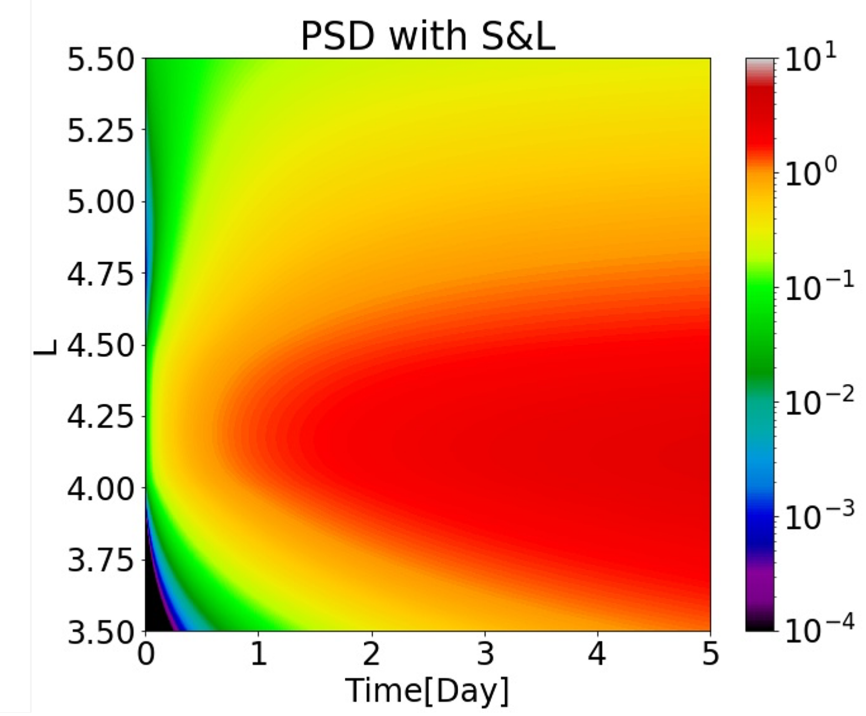
The process of discovering a set of PDEs directly from time-series data collected at a number of spatial locations, can be illustrated in a simple way using the PDE functional identification of nonlinear dynamics (PDE-FIND) algorithm [*Brunton et al.,* 2016; *Rudy et al.,* 2017].



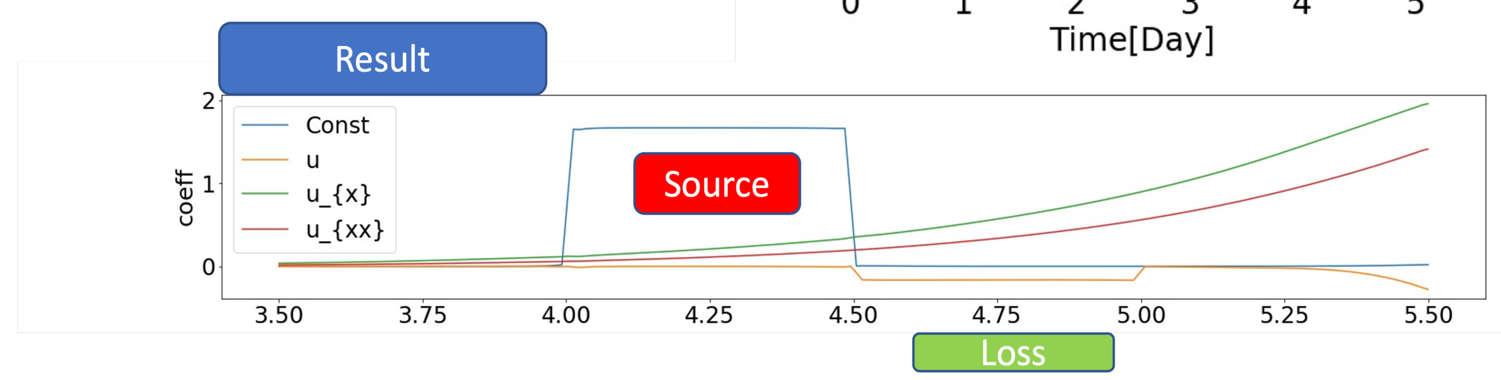
Source and Loss region identification

our method could identify source and loss term using group sparsity

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} D_{LL} L^{-2} \frac{\partial f}{\partial L}$$
$$D_{LL} = 10.0^{(0.506*kp - 9.325)} L^{10} / 86400$$
$$\frac{\partial f}{\partial t} = D_{LL} \frac{\partial^2 f}{\partial L^2} + L^2 \frac{\partial f}{\partial L} \frac{\partial D_{LL} L^{-2}}{\partial L} + S - L$$
$$L = \begin{cases} \frac{f}{\tau} & 4.5 < L < 5 \\ 0 & \text{other} \end{cases}$$
$$S = \begin{cases} const & 4 < L < 4.5 \\ 0 & \text{other} \end{cases}$$



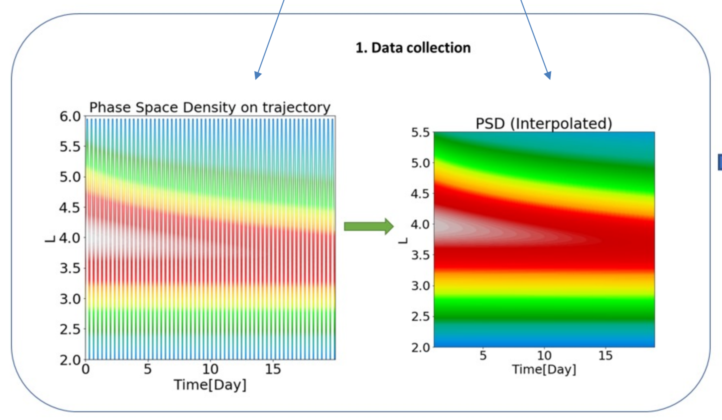
Result



The general flow of data-driven discovery in PSD using integral formula

We performed pilot studies, based on simple simulations of the radiation belts using the reduced Fokker-Planck equation

A machine learning model would be created to help interpolated the PSD data from observation



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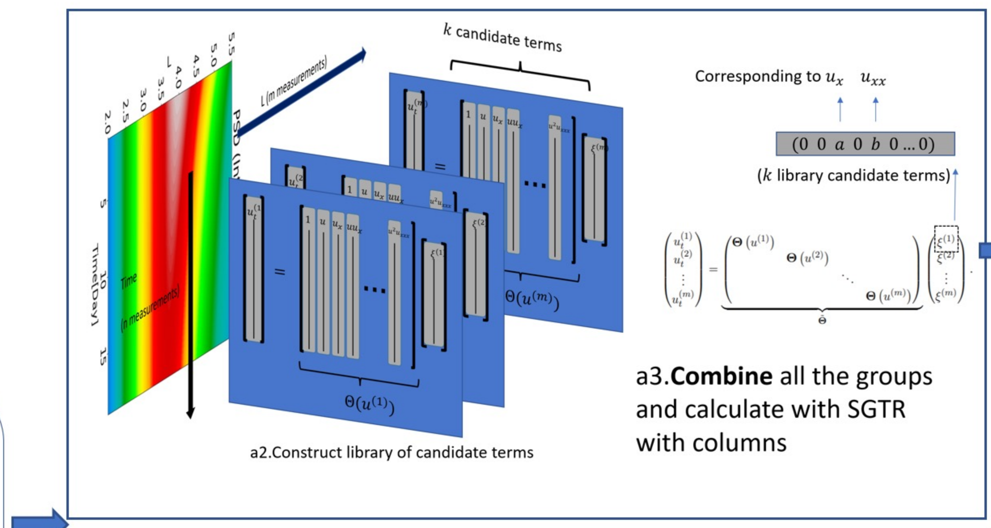
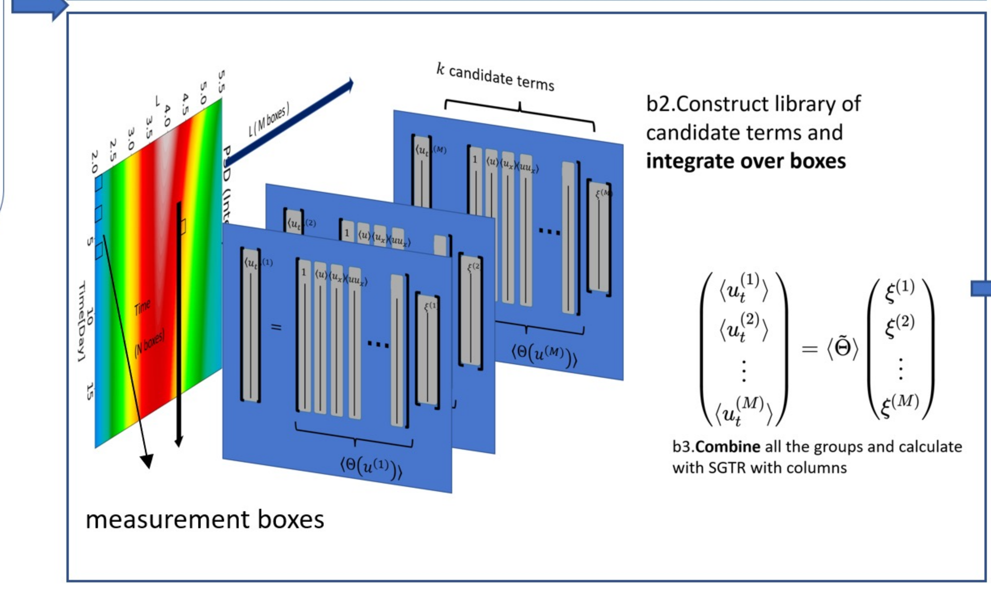
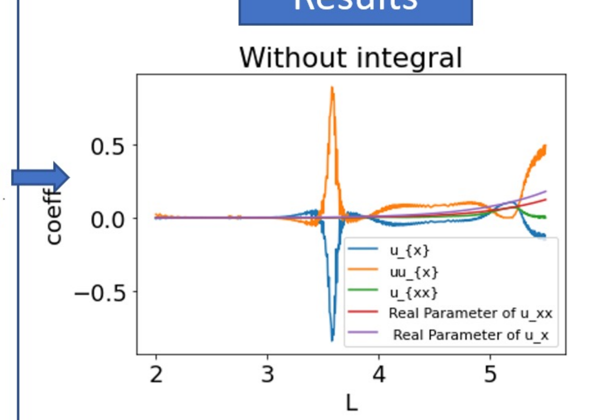
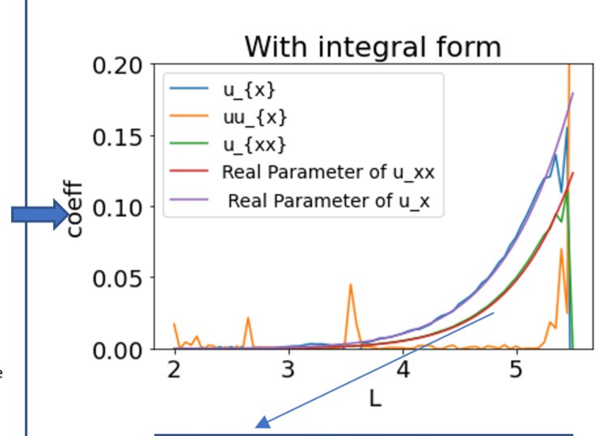




Results

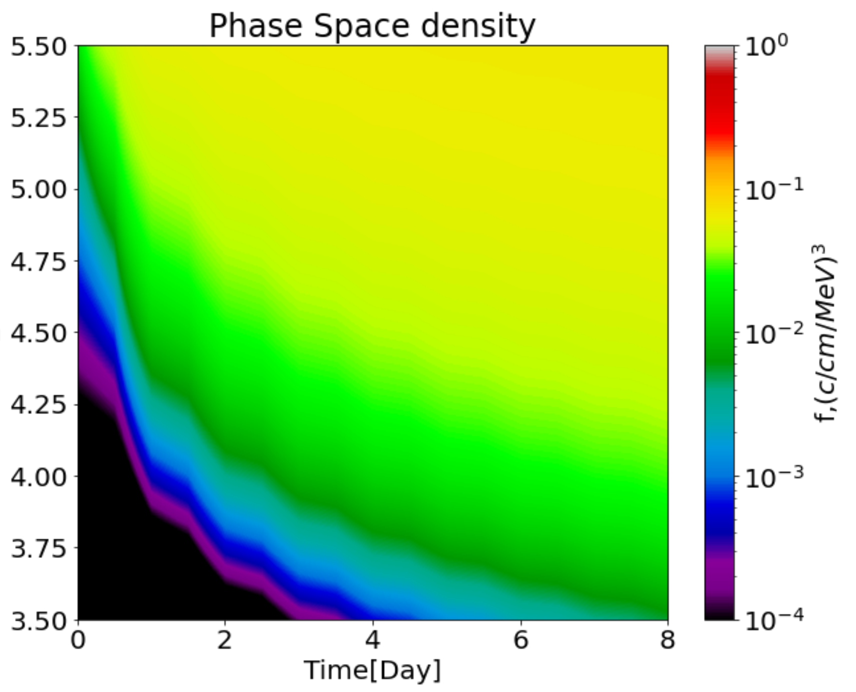


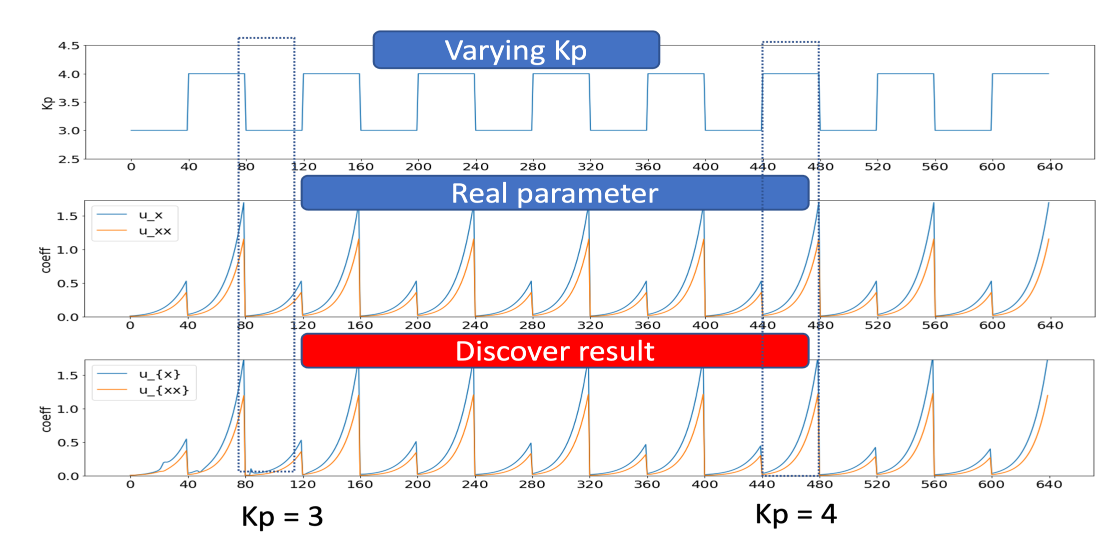


Successfully discover the PDE!

Capture different dynamics

Our method could identify the parametric PDE which is very common in radiation belt dynamics





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Reference

Rudy, S. H., Brunton, S. L., Proctor, J. L., & Kutz, J. N. (2017). Data-driven discovery of partial differential equations. *Science Advances*, *3*(4), e1602614.

Alves, E. P., & Fiuza, F. (2020). Data-driven discovery of reduced plasma physics models from fully-kinetic simulations. *ArXiv*.

Brunton, S. L., Proctor, J. L., & Kutz, J. N. (2016). Discovering governing equations from data by sparse identification of nonlinear dynamical systems. *Proceedings of the National Academy of Sciences*, *113*(15), 3932–3937.