Chaos and spontaneous stochasticity: two sides of (un)predictability

Tommaso ALBERTI

National Institute for Astrophysics, Rome, Italy

tommaso.alberti@inaf.it

CHAOS

$$\begin{cases} \dot{x} = s\left(-x + y\right) \\ \dot{y} = \left(rx - y - xz\right) \\ \dot{z} = \left(xy - bz\right) \end{cases}$$

3 modes, single-scale (large-scale) description

The separation of two initially close trajectories exponentially diverges at $t \to \infty$

λ : **Lyapunov exponents**

SPONTANEOUS STOCHASTICITY

$$\frac{\partial\left(\nabla^2\Psi\right)}{\partial t} = -J\left(\Psi, \left(\nabla^2\Psi\right)\right)$$

21 modes, multi-scale description

The separation of two initially close trajectories continues to be indistinguishable at $t \to \infty$

t^* : **predictable chaos**

How to track the predictability of the geospace during geomagnetic storms and substorms and connect them to solar wind variability?

We use the extremal index (Süveges, 2007) that measures the inverse of the average persistence time around a given state in a region of the phase space (Faranda, 2017,2019)

- It ranges between 0 and 1 and can be related to the metric entropy H corresponding to degree of loss of information (like the Kolmogorov entropy)
- The predictable chaos t* is the inverse of the metric entropy and it measures the predictability horizon time-by-time

SuperMAG indices

Monitoring the dynamical properties and the states corresponding to the dynamics of the auroral electrojets and the ring current as well as their shared properties and coupling

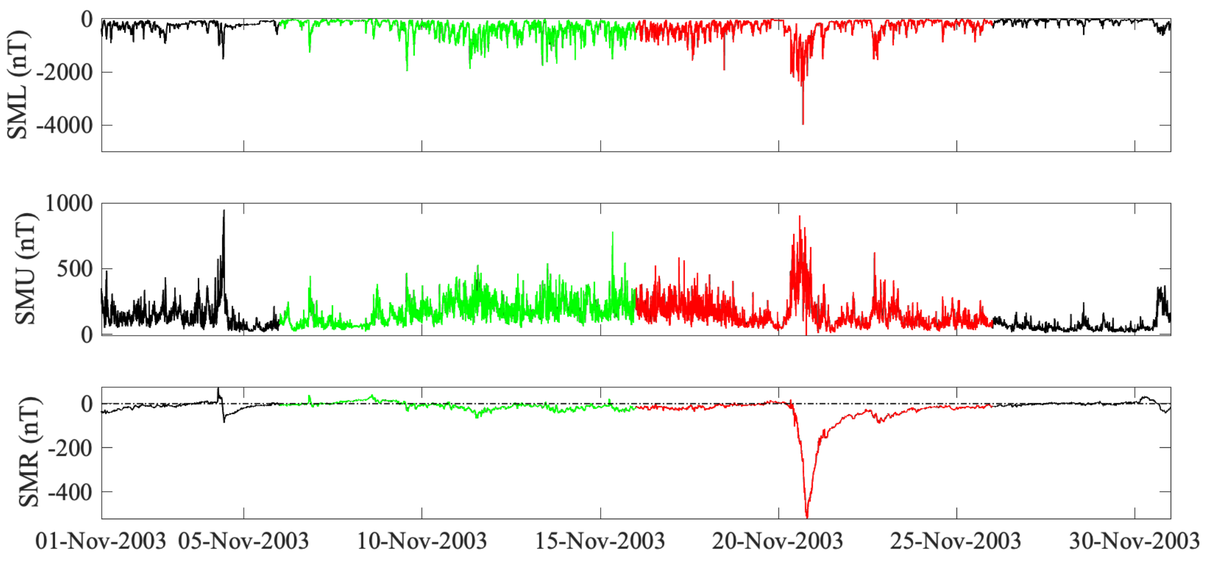


Figure 1. The time series of the three selected geomagnetic indices: SML (top), SMU (middle), and SMR (bottom). Green and red lines refer to two 10-days time intervals corresponding to the quiet– and the storm–time periods, respectively. The horizontal dashed–dotted gray line in the bottom panel shows the zero–level for SMR.

PREDICTABLE CHAOS

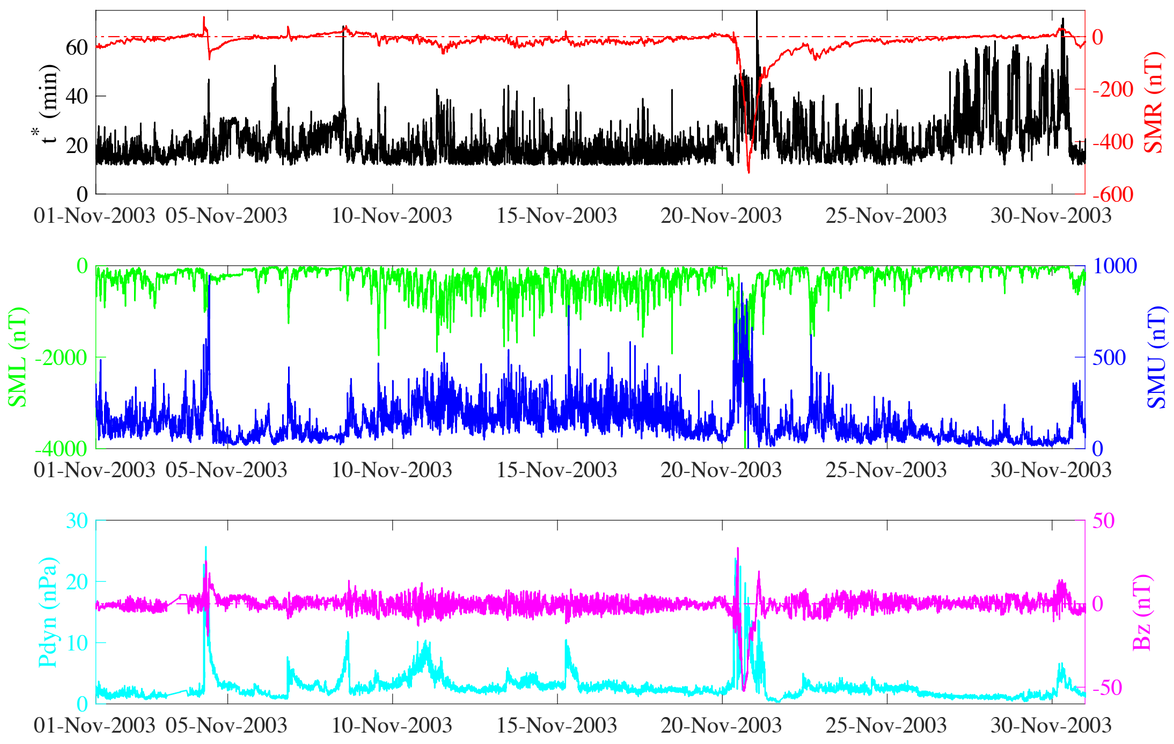
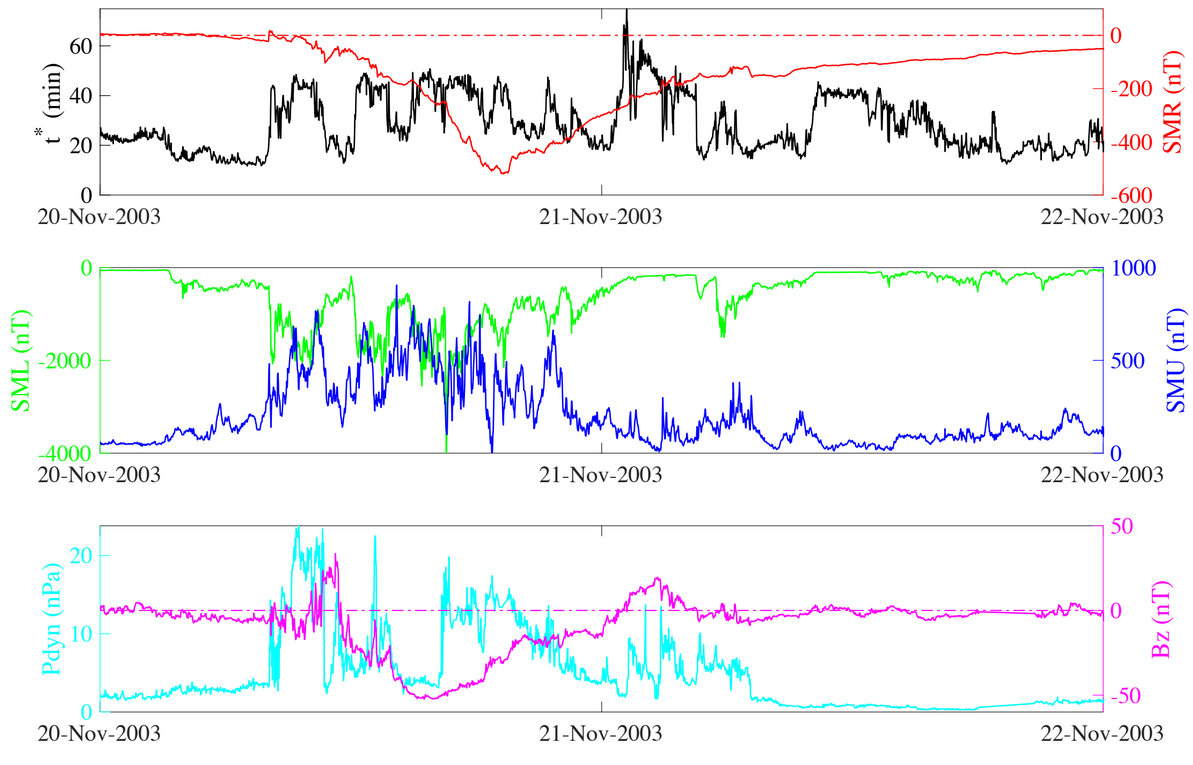


Figure 2. The time series of the predictability time t* (black line), the SMR index (red line), the SML index (green line), the SMU index (blue line), the dynamic pressure (cyan line) and the Bz component (magenta line).



- A relation between the solar wind dynamic pressure and the predictability time t*
- This connection seems to persist also during the main phase of the geomagnetic storm, almost independently on the temporal behavior of the SMR index
- When the auroral activity is low, Bz turned off from its southern orientation (Bz<0), an increase to higher values of t* closely related to the dynamic pressure
- The predictability time is mainly dominated by the auroral activity during the initial, the main, and the early recovery phases of the geomagnetic storm, while during later stages of the recovery phase t* is mainly driven by processes related to the low-latitude ring current activity, closely connected to dynamic pressure fluctuations when the magnetosphere returned to its closed form
- Later on, when the dynamic pressure returned to its standard value, Bz mostly fluctuates around 0 nT with low-amplitude fluctuations, the auroral activity is low, the predictability time tends to return to its pre-storm level (~20 min) in close connection with the end of the recovery phase of the storm (SMR~0 nT).

Data-driven/ML performance could be measured in terms of time-by-time predictability horizon