Dynamic Tomographic Estimation of Global Exospheric Hydrogen Density and its Response to a Geomagnetic Storm

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September 19, 2019
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***Everyone is a genius**. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid.*

- Albert Einstein

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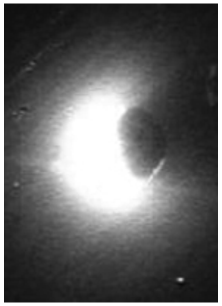
Knowledge of exospheric H density is important but conventional estimation techniques are limited.

What is the topic of study?

- Atomic hydrogen (H) located at the outermost layer of the Earth's atmosphere, resonantly scatters solar Lyman-alpha (121.6nm) radiation



[1]

Why do we need to study this topic?

- To understand various solar-terrestrial interactions such as ring current decaying rate, plasmaspheric refilling as well as evaluate the permanent H escape.

[2], [3]

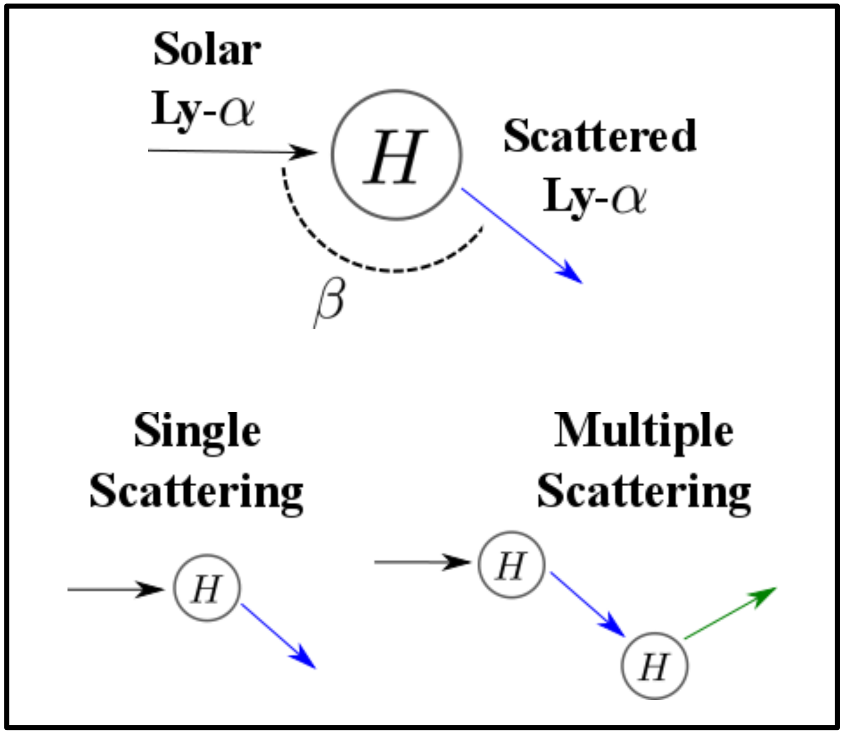
How can we measure the H density?

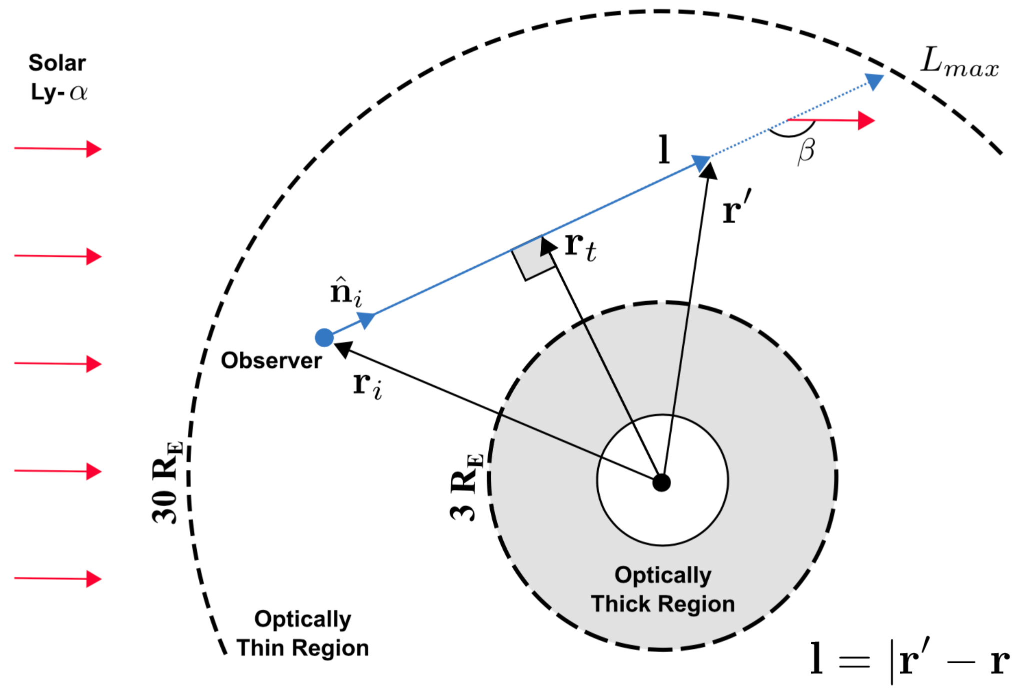
- Direct (in situ) sensing vs. remote sensing.

Image sources: [1] NASA Apollo 16 Mission, [2] https://commons.wikimedia.org/wiki/File:AncientMars.jpg , [3] http://pics-about-space.com/

Main Goal: Generate a remote sensing technique to estimate the Time-dependent, 3-D Hydrogen density distribution in the exosphere.

Hydrogen density estimation leverages the linearity of the optically thin emission model (>3R_E)



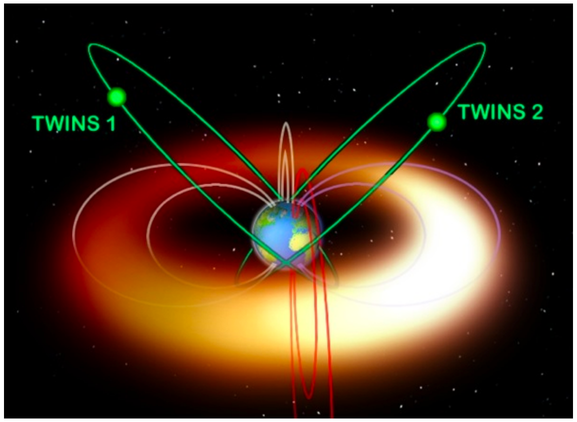


$$I(\mathbf{r}, \hat{\mathbf{n}}, t) = \frac{g^*(t)}{10^6} \int_0^{Lmax} n_H(l)\Psi(\beta)dl + I_{IP}(\hat{\mathbf{n}}, t)$$

- $I(\mathbf{r}, \hat{\mathbf{n}}, t)$: emission intensity (measured) [R] — LAD/TWINS
- $g^*(t)$: Ly-alpha resonant scattering rate (measured) [photons.s^{-1}] — SEE/TIMED
- $n_H(l)$: emitter H density (unknown) [cm^{-3}]
- $\Psi(\beta)$: scattering phase function (known)
- $I_{IP}(\hat{\mathbf{n}}, t)$: interplanetary Background (measured) [R] — SOHO/SWAN

Example of technique feasibility using the NASA's TWINS mission data (static reconstruction)

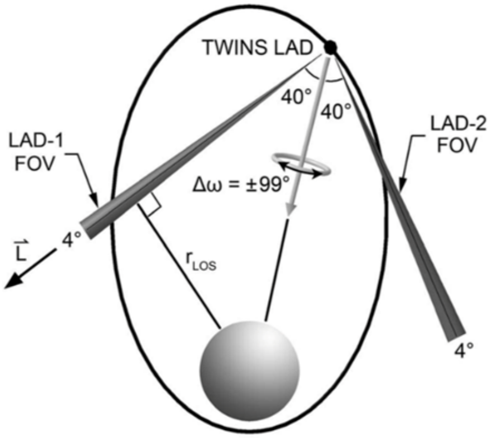
- NASA's Two Wide-angle Imaging Neutral-atom Spectrometers (TWINS) mission provides the capability for **stereoscopically imaging the magnetosphere.**



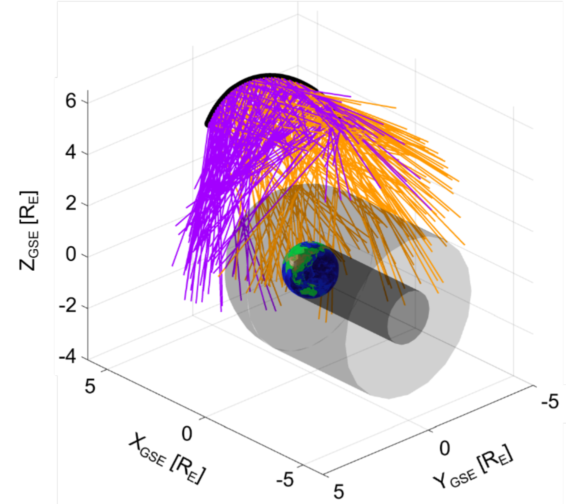
Source: TWINS SWRI website

- Each TWINS1/2 has two **Lyman-alpha detectors (LAD)**, optical sensors.

- The selected data in this study is from **11 June 2008**, in order to compare results with those reported by Bailey et al., [2011]



Source: [Bailey et al., 2011]

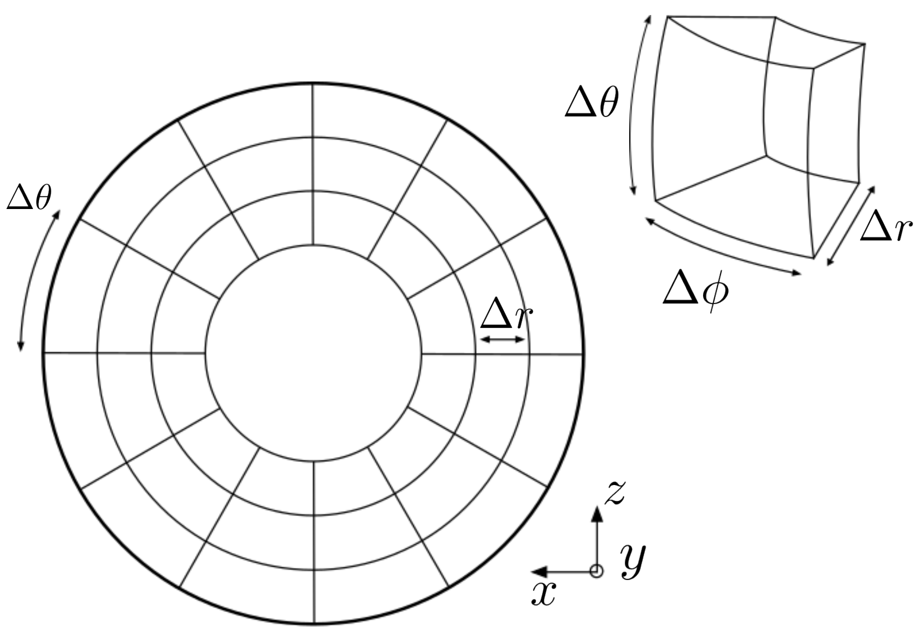


- Since it is quiet-time we assume a **temporally-static** H exosphere.

Discretization of the exospheric volume of interest yields an algebraic linear system.

$$I(\mathbf{r}_i, \hat{\mathbf{n}}_i) = \frac{g^*(\mathbf{r}_i)}{10^6} \int_0^{Lmax} n_H(l)\Psi(\hat{\mathbf{n}}_i)dl + I_{IP}(\hat{\mathbf{n}}_i)$$

- **Step 1:** Discretize region into J spherical voxels.



- **Step 2:** Project unknown density function onto J orthonormal basis functions.

$$n_H(r') = \sum_{j=1}^{J} x_j \delta_{H_j}(r'),$$

$$\delta_{Hj}(r') = \begin{cases} 1 & \text{if } r' \in V_j \\ 0 & \text{else} \end{cases}$$

- **Step 3:** Rewrite i^{th} measurement of intensity as a linear equation.

$$y(\mathbf{r}_i, \hat{\mathbf{n}}_i) = \sum_{j=1}^{J} [\frac{g^*(\mathbf{r}_i)}{10^6} \Psi(\hat{\mathbf{n}}_i) \int_0^{Lmax} \delta_{H_j}(l)dl] x_j$$

$$\mathbf{y} = L\mathbf{x}$$

$\mathbf{y} \in \mathbb{R}^M$

$\mathbf{x} \in \mathbb{R}^J$

$L \in \mathbb{R}^{M \times J}$

Solving the estimation problem requires the use of more complex techniques such as regularization

- Observation matrix $L \in \mathbb{R}^{M \times J}, \; M \gg J$ and **is not full column rank** (Voxels with out LOS through them).

$$\hat{\mathbf{x}} = \underset{x \geq 0}{\operatorname{argmin}} \, \Phi(\mathbf{x})$$

- Regularization techniques are necessary to obtain a solution.

$$\Phi(\mathbf{x}) = ||L\mathbf{x} - \mathbf{y}||_2^2 + \lambda RRPE(\mathbf{x})$$

Cost Func. — Data misfit term — Regularization term

- The selected regularization method is **Regularized Robust Positive Estimation**.

$$\lambda RRPE(\mathbf{x}) = \lambda_r ||\mathbf{x}||_{D_r} + \lambda_\phi ||\mathbf{x}||_{D_\phi} + \lambda_\theta ||\mathbf{x}||_{D_\theta}$$

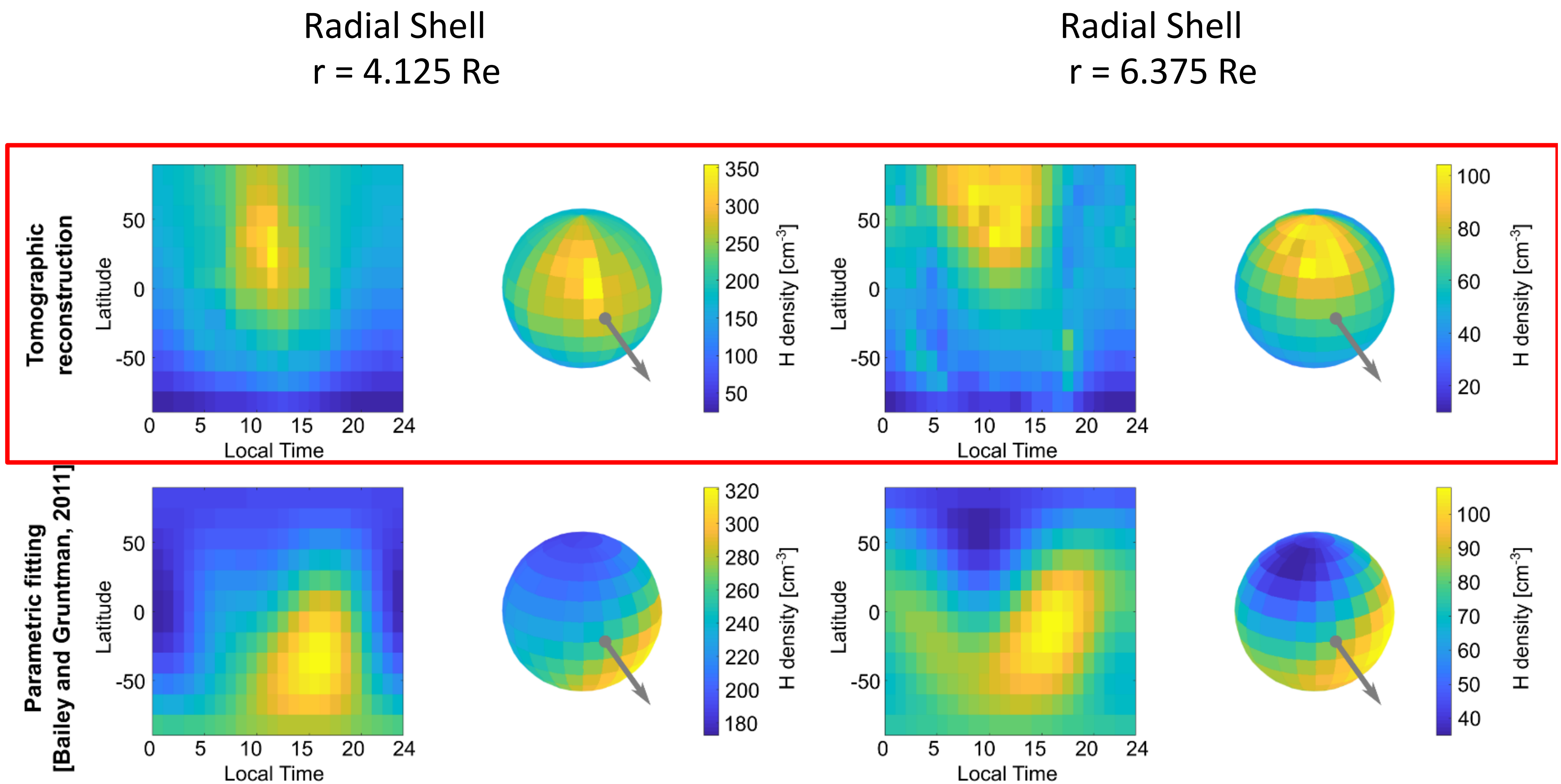
Radial dim. — Azimuthal dim. — Polar dim.

- Includes prior knowledge of **physical structure of the Hydrogen density distributions** for each dimension.

$$||\mathbf{x}||_{D_r} = \mathbf{x}^T D_r^T D_r \mathbf{x}$$

Discrete matrix form of 1st and 2nd derivatives

$$D_r \approx \partial^2 / \partial r^2$$
$$D_\phi \approx \partial / \partial \phi$$
$$D_\theta \approx \partial / \partial \theta$$



[Cucho-Padin & Waldrop, JGR, 2018]

Space-state framework approach for "*dynamic tomography*" and Kalman Filter as a solver

As exospheric H densities are prone to be dynamic during storm-time, we use the state-space model as a means for time-varying estimation:

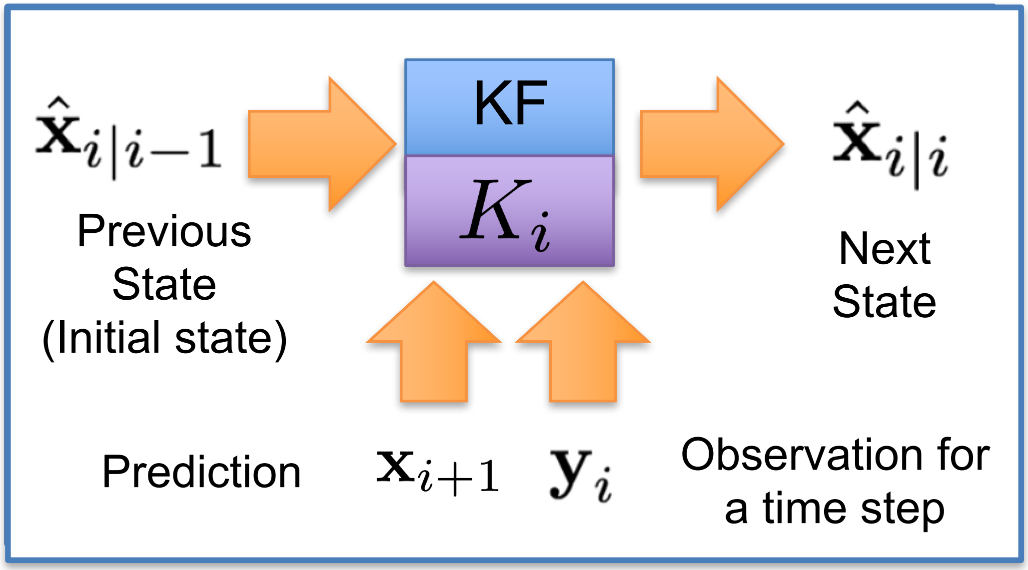
Measurement equation:

$$\mathbf{y}_i = H_i \mathbf{x}_i + \mathbf{v}_i$$

Model evolution equation:

$$\mathbf{x}_{i+1} = F_i \mathbf{x}_i + \mathbf{u}_i$$

Kalman Filter as solver



Inclusion of regularization terms

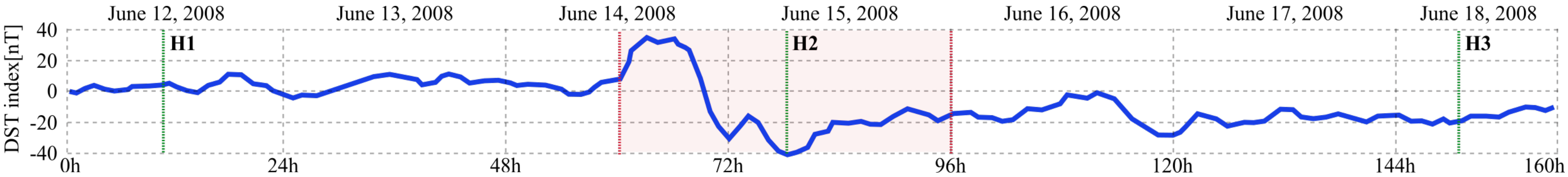
$$\begin{bmatrix} \mathbf{y}_i \\ 0 \end{bmatrix} = \begin{bmatrix} H_i \\ D_i \end{bmatrix} \mathbf{x}_i + \begin{bmatrix} \mathbf{v}_i \\ \mathbf{w}_i \end{bmatrix}$$

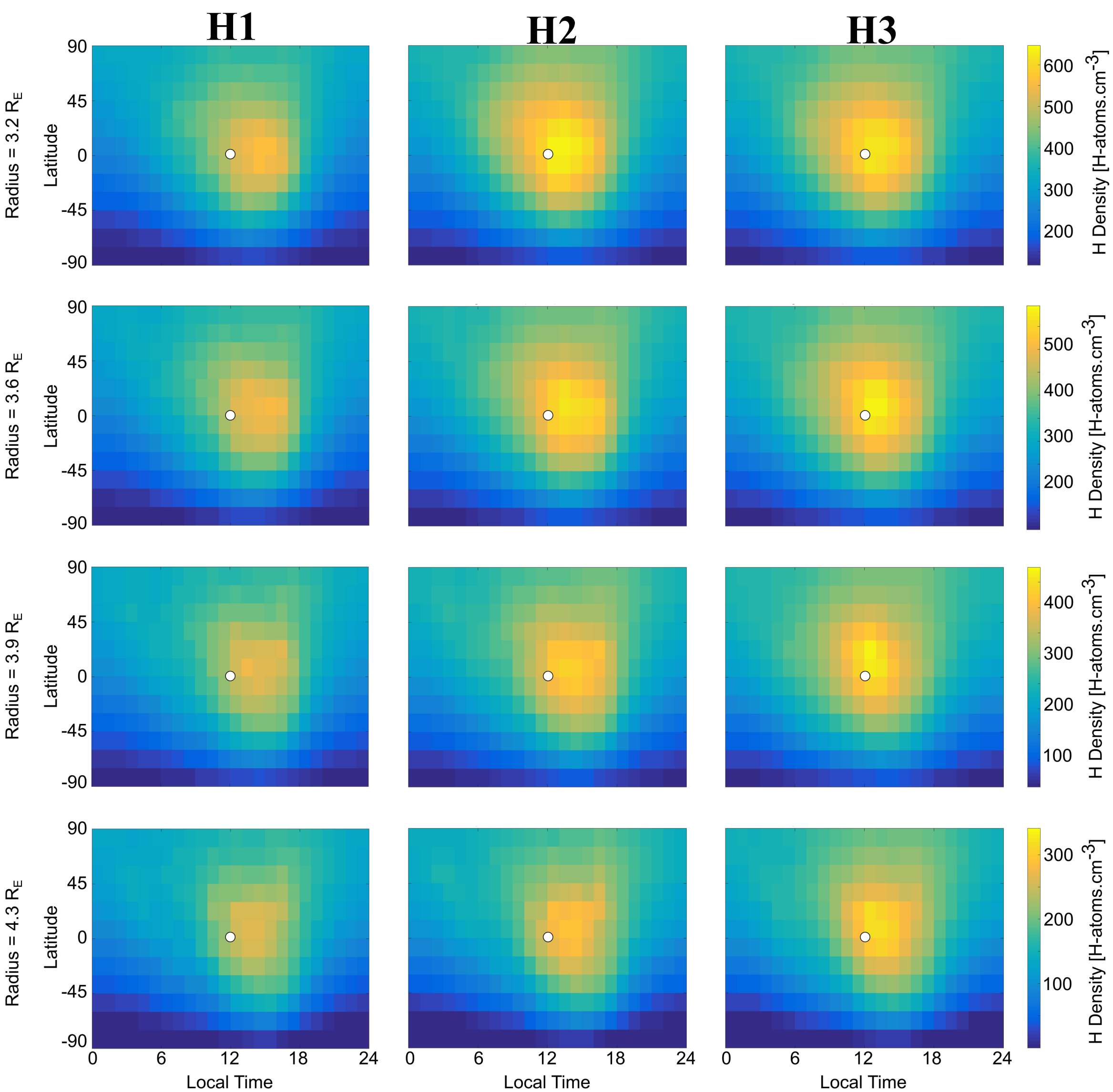
$$\mathbf{y}'_i = H'_i \mathbf{x}_i + v'_i$$

$$R'_i \triangleq \mathbb{E}[\mathbf{v}'_i (\mathbf{v}'_i)^T] = \begin{bmatrix} R_i & 0 \\ 0 & \lambda_i^{-1} I \end{bmatrix}$$

Dynamic tomographic estimation connected to the LMMSE estimation

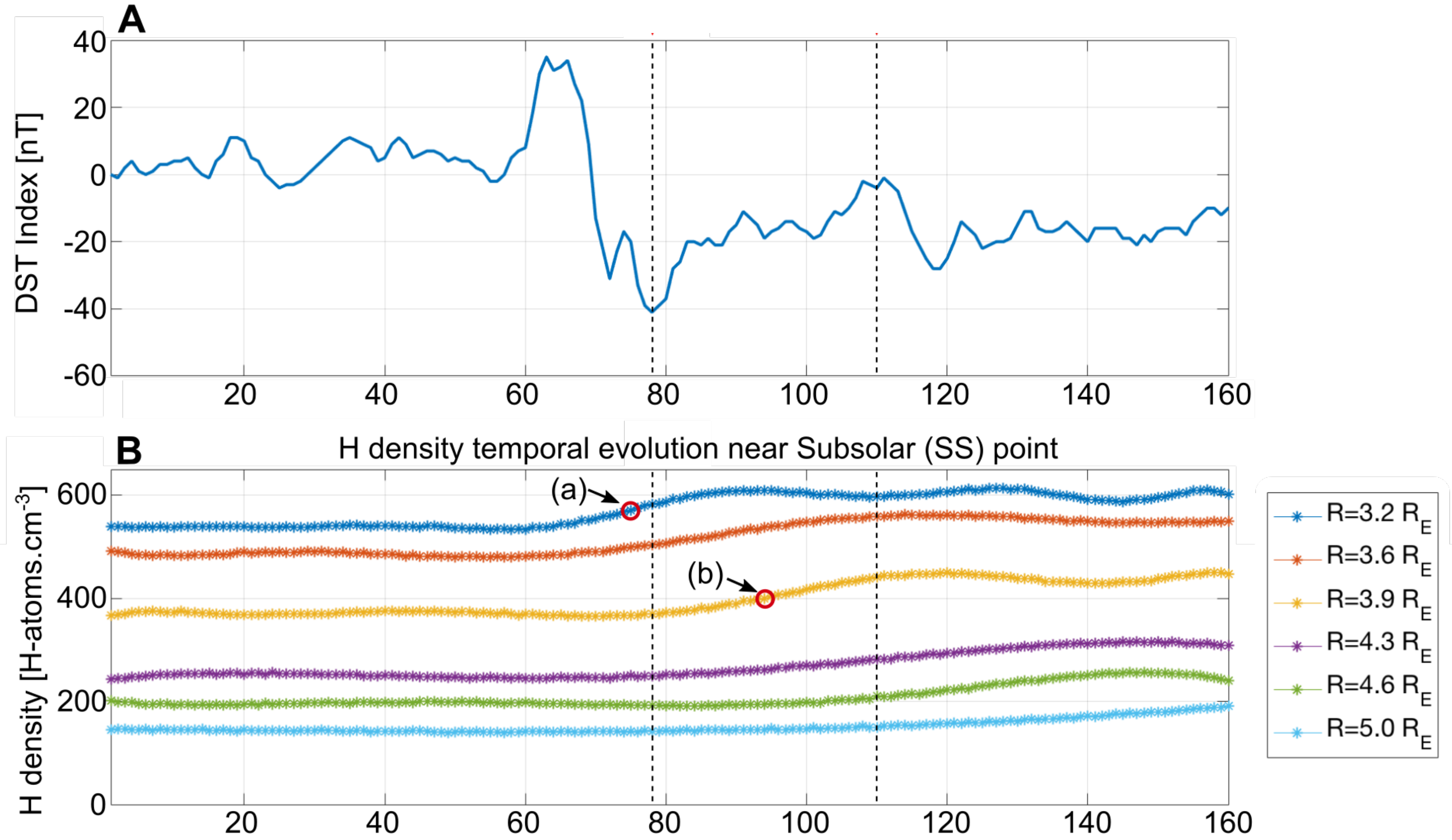
$$\hat{\mathbf{x}}^d_{i|i} = \underset{\mathbf{x}_i}{\text{argmin}} \, ||\mathbf{y}'_i - H'_i \mathbf{x}_i||^2_{R_i'^{-1}} + ||\mathbf{x}_i - \hat{\mathbf{x}}_{i|i-1}||^2_{P^{-1}_{i|i-1}} + \lambda_\phi ||D_\phi \mathbf{x}_i||_2^2 + \lambda_\theta ||D_\theta \mathbf{x}_i||_2^2 + \lambda_r ||D_r \mathbf{x}_i||_2^2$$





- ⦿ Using KF we have performed 160 dynamic reconstructions during the storm occurred in 15, June, 2008.

- ⦿ Hydrogen density enhancements during the storm development. Such increments are then translate to higher altitudes with certain delay which suggest a vertical transport or upwelling.



- Hydrogen density enhancement at 3.2 Re is equal to ~15%.

- In the subsolar point, calculations between 3.2Re and 3.9 Re profiles result in a exospheric wind of ~60m/s.

[Cucho-Padin & Waldrop, GRL, 2019]

Summary

- Dynamic tomography based on TWINS observations shows that H density increases abruptly in response to the geomagnetic storm on 15 June, 2008. The increment rate and its magnitude varying with distance from Earth.

- Density increases begin soonest in the innermost exospheric region in the reconstruction (3.2 RE) and reach a peak density fastest there. Overall density enhancements of **~15%** are observed at **3.2 RE**. Recovery to pre-storm values is very slow.

- Also, analysis of the radial structure for the subsolar point yielded a **~60 m/s** wind in vertical direction.

- Further work :
  1. Conduct similar experiments during a strong geomagnetic storm.
  2. Use of tomographically-reconstructed H densities in ring current and plasmasphere analysis during storm-time.

References

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